

OBJECTIVE QUESTIONS

1. Which of the following is the value of $\lim_{x \rightarrow 2} \left(\frac{x^2 - 4}{x^2 + 4} \right)$?
- (a) 1 (b) 0 (c) $\frac{-1}{2}$ (d) ∞
2. Which of the following is the value of $\lim_{x \rightarrow \infty} \left(\frac{4 - x^2}{x^2 - 1} \right)$?
- (a) 1 (b) 0 (c) -4 (d) -1
3. What is the value of $\lim_{x \rightarrow 1} \left(\frac{x^2 - x}{2x^2 - 2x} \right)$?
- (a) -2 (b) -1 (c) 1 (d) does not exist
4. What is the value of $\lim_{x \rightarrow 1} \left(\frac{x^2 - x}{2x^2 - 2x} \right)$?
- (a) -2 (b) -1 (c) $\frac{1}{2}$ (d) does not exist
5. What is the value of $\lim_{x \rightarrow 4} \left(\frac{x^2 - 2x - 8}{x - 4} \right)$?
- (a) 4 (b) -4 (c) 6 (d) -6
6. A function defined by $f(x) = \frac{x - |x|}{x}$, $x \neq 0$, and $f(0) = 2$, the which of the following is correct?
- (a) f is continuous at $x = 1$ (b) f is continuous at $x = 0$
(c) f is continuous at $x = -1$ (d) f is continuous at $x = 2$.
7. If $f(x) = \begin{cases} 1 + kx & x \leq 3 \\ 1 - kx^2 & x > 3 \end{cases}$ is continuous at $x = 3$, then which of the following is the value of k ?
- (a) 0, 1 (b) 0 (c) 1, -1 (d) 2

8. If $f(x) = \begin{cases} x^2 - 1 & x \neq 1 \\ 4 & x = 1 \end{cases}$, then which of the following is true?

1. $\lim_{x \rightarrow 1} f(x)$ exists. 2. $f(1)$ exist. 3. f is continuous at $x = 1$.

(a) 1 is true

(b) 2 is true

(c) 1 and 2

(d) 1, 2, 3 true

9. If $f(x) = \begin{cases} \frac{x^2 - x}{2x} & x \neq 0 \\ f(0) = k \end{cases}$ is continuous at $x = 0$, then which of the following is the value of k ?

(a) -1

(b) $-\frac{1}{2}$

(c) 0

(d) 1

10. If $\lim_{x \rightarrow a^-} f(x)$ and $\lim_{x \rightarrow a^+} f(x)$ both exist but not equal, then which of the following is true?

(a) f has removable discontinuity

(b) f has discontinuity of first kind

(c) f has limit

(d) f has discontinuity of second kind

Answer:

1. (b)

2. (d)

3. (b)

4. (c)

5. (c)

6. (b)

7. (c)

8. (c)

9. (b)

10. (b)

Objective Questions

1. The derivative of a function $f(x)$ is

(a) $f'(x) = \lim_{\Delta x \rightarrow 0} \left(\frac{f(x + \Delta x) + f(x)}{\Delta x} \right)$ (b) $f'(x) = \lim_{\Delta x \rightarrow 0} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right)$

(c) $f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x}$ (d) $f'(x) = \frac{f(x + \Delta x) + f(x)}{\Delta x}$

2. If $y = \frac{u}{v}$ then $\frac{dy}{dx}$ is

(a) $\frac{v \frac{du}{dx} + u \frac{dv}{dx}}{v^2}$

(b) $\frac{u \frac{dv}{dx} - v \frac{du}{dx}}{u^2}$

(c) $\frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

(d) $\frac{u \frac{dv}{dx} - v \frac{du}{dx}}{v^2}$

3. If $y = 2^x$ then $\frac{dy}{dx}$ is

(a) $x(2^{x-1})$

(b) $\frac{2^x}{\ln x}$

(c) $2^x \ln(2)$

(d) none of these

4. If $x = at^2$ and $y = 2at$ then $\frac{dy}{dx}$ is

(a) $1/t$

(b) $-1/t^2$

(c) $-2/t$

(d) $2/t$

5. If $y^3 = x^2$ then $\frac{dy}{dx}$ is

(a) $\frac{2x}{y^3}$

(b) $\frac{2x}{3y^2}$

(c) $\frac{x^2}{3y^2}$

(d) $\frac{x^2}{3y}$

6. The derivative of $\frac{1 - \cos 2x}{\sin 2x}$ is

(a) $\sec^2 x$

(b) $\tan x$

(c) $\ln(\cos x)$

(d) $\ln(\sin x)$

7. If $f(x) = \frac{1}{2}(e^x + e^{-x})$ then $f'(0)$ is

(a) 1

(b) 0

(c) -1

(d) 2

8. If $f(1) = -1$, $f'(1) = 1$, $g(1) = 3$ and $g'(1) = 4$ then value of $\frac{d(fg)}{dx}$ at $x = 1$ is
- (a) 1 (b) 0 (c) -1 (d) 2
9. The 4th derivative of function $x^4 + 3x - 9$ is
- (a) 4 (b) 0
(c) 24 (d) none of these
10. If $f(x) = \sqrt{3 - 2x}$ then $f'(x)$ is
- (a) $\frac{-2}{\sqrt{3 - 2x}}$ (b) $\frac{1}{\sqrt{3 - 2x}}$ (c) $\frac{2}{\sqrt{3 - 2x}}$ (d) $\frac{-1}{\sqrt{3 - 2x}}$
11. If $y = \ln(\sin(\cos x))$ then $\frac{dy}{dx}$ is
- (a) $\frac{1}{\sin(\cos x)}$ (b) $\frac{\cos x}{\sin(\cos x)}$
(c) $\frac{\cos(\cos x)}{\sin(\cos x)}$ (d) $-\sin x \cot(\cos x)$
12. If $x^2 + y^2 = 5$ then $\frac{dy}{dx}$ is
- (a) x/y (b) y/x (c) $-x/y$ (d) $-y/x$
13. Which of the following statement is correct;
- (a) Every continuous function is differentiable
(b) Every differentiable function is continuous
(c) Every continuous function may or may not be differentiable
(d) Every differentiable function may or may not be continuous
14. In curve $y = x^4 - 10$, if x changes from 2 to 1.99 then change in y is
- (a) 0.32 (b) 0.032 (c) 5.68 (d) 5.968
15. The change in area of square whose side is increased from 1 cm to 1.01 is
- (a) 0.0001 cm^2 (b) 0.01 cm^2
(c) 0.02 cm^2 (d) 0.2 cm^2
16. The points at which tangents to curve $y = x^3 - 12x + 18$ are parallel to x -axis are
- (a) $(2, -2)$ and $(-2, -34)$ (b) $(2, 34)$ and $(-2, 0)$
(c) $(0, 34)$ and $(-2, 0)$ (d) $(2, 2)$ and $(-2, 34)$

17. Equation of tangent of curve $y = x^2 - x$ at $x = 1$ is
 (a) $x - y = 1$ (b) $x + y = 1$
 (c) $x - y + 1 = 0$ (d) $x + y + 1 = 0$
18. Slope of tangent to circle $x^2 + y^2 = 100$ at point $(-6, 8)$ is
 (a) $3/4$ (b) $-3/4$ (c) $4/3$ (d) $-4/3$
19. The point on curve $y = x^2 - x$ where slope of tangent is -1 , is
 (a) $(1, 0)$ (b) $(0, 1)$ (c) $(1, 1)$ (d) $(0, 0)$
20. Rolle's Theorem asserts that
 (a) Existence of at least one point on given interval, tangent at that point is parallel to x -axis.
 (b) Existence of at least one point on given interval, tangent at that point is perpendicular to x -axis.
 (c) Existence of at least one point on given interval, tangent at that point is parallel to y -axis.
 (d) None of above.
21. Mean Value Theorem asserts that
 (a) Existence of at least one point on given interval, tangent at that point is parallel to x -axis.
 (b) Existence of at least one point on given interval, tangent at that point is perpendicular to x -axis.
 (c) Existence of at least one point on given interval, tangent at that point perpendicular to the chord joining the end points of that interval
 (d) Existence of at least one point on given interval, tangent at that point is parallel to chord joining end of points of that interval.
22. For the function $f(x) = x^2 - 2x$ in $[1, 2]$, the value of c for Mean Value Theorem is
 (a) 1 (b) $1/2$ (c) $2/3$ (d) $3/2$
23. For the function $f(x) = \sin x$ in $[0, \pi]$, the value of c for Rolle's Theorem is
 (a) 0 (b) $\pi/2$ (c) π (d) $\pi/4$
24. The necessary condition of Mean Value Theorem for function $f(x)$ is
 (a) continuous in $[a, b]$ (b) differentiable in (a, b)
 (c) both (a) and (c) (d) none of these

Answer:

- | | | | | | | | |
|---------|---------|---------|---------|---------|---------|---------|---------|
| 1. (b) | 2. (c) | 3. (c) | 4. (a) | 5. (b) | 6. (a) | 7. (b) | 8. (c) |
| 9. (b) | 10. (d) | 11. (d) | 12. (c) | 13. (b) | 14. (a) | 15. (c) | 16. (b) |
| 17. (b) | 18. (b) | 19. (d) | 20. (a) | 21. (a) | 22. (d) | 23. (b) | 24. (c) |

OBJECTIVE QUESTIONS

- The slope of the tangent to the curve $x^2 - 2x + 2$ at $(\frac{1}{2}, 0)$ is
(a) -1 (b) 1 (c) 0 (d) $\frac{1}{2}$
- The inclination with the x-axis of the tangent to the curve $x^2 + y^2 = 36$ at $(0, 6)$ is given by
(a) π (b) $\frac{3\pi}{2}$ (c) 0 (d) $\frac{\pi}{2}$
- The function $f(x) = x^2 - 2x$ is increasing in the interval.
(a) $x > 1$ (b) $x \leq -1$ (c) $x \geq -1$ (d) $x \leq 1$
- The minimum value of the function $f(x) = x^3 - 6x^2 + 9x + 12$ is
(a) 12 (b) 10 (c) -10 (d) -2
- The maximum value of the function $f(x) = 2x^3 - 15x^2 + 36x + 5$ is
(a) 13 (b) 3 (c) -3 (d) 33
- The sum of two non zero number is 4. What is the minimum value of the sum of their reciprocals?
(a) $\frac{1}{4}$ (b) 0 (c) $\frac{1}{2}$ (d) 1
- The critical points of the function $f(x) = (x - 2)^{2/3}(2x + 1)$ are
(a) 1 and 2 (b) -1 and 2 (c) 1 only (d) 1 and $-\frac{1}{2}$
- The curve $y = f(x)$ will be concave downward in the interval when it is defined if
(a) $f''(x) = 0$ (b) $f''(x) < 0$ (c) $f''(x) > 0$ (d) $f'(x) < 0$
- The function $f(x) = x^3 - 3x^2 + 1$ at $x = -2$ is
(a) concave downward (b) concave upward
(c) point of inflection (d) none
- The cost $c(x) = 20 + 2x + 0.5x^2$ then slope of cost is
(a) 0 (b) 1 (c) 2 (d) 0.5

Answer:

- | | | | | |
|--------|--------|--------|--------|---------|
| 1. (a) | 2. (c) | 3. (a) | 4. (a) | 5. (d) |
| 6. (d) | 7. (a) | 8. (b) | 9. (a) | 10. (b) |

OBJECTIVE QUESTIONS

1. The area of the region bounded by the curve $y = x^2$ and $x = 1$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{7}{3}$

2. $\int (2x + 1)^5 dx =$
(a) $\frac{(2x + 1)^6}{12} + C$ (b) $\frac{(2x + 1)^6}{6} + C$
(c) $6(2x + 1)^5 + C$ (d) $12(2x + 1)^5 + C$

3. $\int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}} =$
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

4. $\int_0^2 \frac{2x dx}{\sqrt{x^2 + 4}} =$
(a) $2\sqrt{2}$ (b) 4 (c) $\frac{4\sqrt{2}}{3}$ (d) $2\sqrt{2} - 2$

5. The area bounded by the curve $y = \ln x$, x -axis and the line $x = e$ is
(a) e (b) 1 (c) $\frac{1}{e}$ (d) $1 + \frac{1}{e}$

6. $\int_0^4 \frac{dx}{\sqrt{4-x}} =$
(a) 3 (b) 2 (c) 4 (d) 1

7. $\int_0^1 2x e^{x^2} dx =$
(a) $e - 1$ (b) $e - 2$ (c) e (d) $e - 4$

8. $\int_1^{\infty} \frac{1}{x^4} dx =$
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

9. Area between $y = x$ and $y = x^2$ is
 (a) $\frac{1}{6}$ (b) 1 (c) $\frac{1}{3}$ (d) 2
10. Area under the curve $y = \cos x$, $0 \leq x \leq \frac{\pi}{2}$ is
 (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 1 (d) 2
11. Volume of sphere of unit radius is
 (a) $\frac{2\pi}{3}$ (b) $\frac{4\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{3\pi}{2}$
12. Volume of cone with unit base radius is
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi h}{3}$ (c) $\frac{2\pi h}{3}$ (d) $\frac{\pi h^3}{3}$
13. The average value of $f(x) = x^3$ on $[0, 2]$ is
 (a) 1 (b) 2 (c) 3 (d) 4
14. Approximate value of $\int_0^1 \frac{dx}{1+x^2}$ with $n = 2$. Using Trapezoidal rule
 correct to three places of decimals is
 (a) 0.324 (b) 0.355 (c) 0.785 (d) 0.542
15. Approximate value of $\int_0^1 \frac{dx}{1+x}$ with $n = 4$. Using Simpson's rule
 correct to three places of decimals is
 (a) 0.194 (b) 0.201 (c) 0.432 (d) 0.693
16. The area bounded by parabola $y^2 = 4x$, x -axis and line $x = 1$ is
 (a) $\frac{1}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{8}$
17. $\int_0^{\pi/4} \cos x \, dx$
 (a) $\frac{\pi}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $2 - \sqrt{2}$ (d) $2 + \sqrt{2}$

ANSWER

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (c) | 2. (a) | 3. (d) | 4. (b) | 5. (c) |
| 6. (c) | 7. (a) | 8. (c) | 9. (a) | 10. (c) |
| 11. (b) | 12. (b) | 13. (b) | 14. (c) | 15. (d) |
| 16. (b) | 17. (b) | | | |

Objective Questions

1. The order and degree of a differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^2 = x$ is
(a) 6, 9 (b) 3, 6 (c) 3, 2 (d) 2, 3
2. The degree and order of a differential equation $\left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{1 + \frac{d^2y}{dx^2}}$ is
(a) 2, 2 (b) 1, 2 (c) 2, 4 (d) 4, 2
3. The solution of a differential equation $x + y = \cos^{-1}\left(\frac{dy}{dx}\right)$ is
(a) $x = \tan\left(\frac{x+y}{2}\right) + C$ (b) $y = \sin\left(\frac{x+y}{2}\right) + C$
(c) $x = \tan(x+y) + C$ (d) $y = \sin(x+y) + C$

4. The solution of the differential equation $\tan x \, dy + \tan y \, dx = 0$ is
 (a) $y \tan x + x \tan y = C$ (b) $\sec^2 x + \sec^2 y = C$
 (c) $\sin x + \sin y = C$ (d) $\sin x \sin y = C$
5. The solution of a differential equation $e^{x-y} \, dx + e^{y-x} \, dy = 0$ is
 (a) $e^{xy} = C$ (b) $e^{2x} + e^{2y} = C$
 (c) $x + y = C$ (d) $x - y = C$
6. If $\cot x + \tan y = C$ is a solution of a differential equation then the corresponding differential equation is
 (a) $\frac{dy}{dx} = \cot x + \tan y$ (b) $\frac{dy}{dx} = \cot x - \tan y$
 (c) $\frac{dy}{dx} = \frac{\cos^2 y}{\sin^2 x}$ (d) $\frac{dy}{dx} = \frac{\sin^2 x}{\cos^2 y}$
7. The solution of a differential equation $\frac{dy}{dx} = \frac{y-x}{x}$ is
 (a) $y - x \ln x = cx$ (b) $y + x \ln x = cx$
 (c) $x + y \ln y = cy$ (d) $x - y \ln y = cy$
8. The solution of a differential equation $x \, dy - y \, dx = 0$ is
 (a) $x = cy$ (b) $y = cx$
 (c) $xy = c$ (d) none of these
9. The solution of a differential equation $\frac{dy}{dx} = e^x$ is
 (a) $y = e^x + c$ (b) $y = e^x$ (c) $ye^x = c$ (d) $y = \frac{c}{e^x}$
10. The solution of a differential equation $\frac{dy}{dx} = \frac{1+y}{1+x}$ is
 (a) $1 + y = c(1 + x)$ (b) $1 + x = c(1 + y)$
 (c) $1 + x = c(x + y)$ (d) $1 + x = c(1 + y)$
11. The solution of a differential equation $\frac{dy}{dx} + \cot y = 0$ is
 (a) $e^x = c \cos y$ (b) $e^x = c \sin y$ (c) $e^y = c \cos x$ (d) $e^y = c \sin x$
12. The solution of a differential equation $\frac{dy}{dx} = \frac{y}{x}$ is
 (a) $y = \frac{c}{x}$ (b) $y = c \ln x$ (c) $y = cx$ (d) $\ln y = cx$

13. The solution of a differential equation $\frac{dy}{dx} = 2x + 5$ is

(a) $y = x^2 + 5x + 1$

(b) $y = x^2 + 5x$

(c) $y = x^2 + 5x + c$

(d) $y = x^2 + 5x + cx$

14. The solution of a differential equation $\frac{dy}{dx} = \frac{ax + b}{cy + d}$ is

(a) $cy + d = ax + b + k$

(b) $cy^2 + d^2 = ax^2 + b^2$

(c) $cy^2 + 2d = ax^2 + 2b$

(d) $cy^2 + 2dy = ax^2 + 2bx + k$

ANSWER:

1. (c)

2. (d)

3. (a)

4. (c)

5. (b)

6. (c)

7. (b)

8. (b)

9. (a)

10. (d)

11. (a)

12. (c)

13. (c)

14. (d)

OBJECTIVE QUESTIONS

1. The system of equations $x - 2y + 5 = 0$ and $2x - 4y + 7 = 0$ is
(a) consistent (b) inconsistent
(c) consistent with unique solution (d) Neither
2. The inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$
(a) exists (b) does not exist (c) doubtful
3. The system of equations $x - y = 2$, $3x - 3y = 6$ represents
(a) Parallel lines (b) coincident lines
(c) intersecting lines (d) none
4. The graph of $3x + 2y \geq 6$ contains the point
(a) (5, 3) (b) (0, 0) (c) (1, 1) (d) (-2, 1)

5. Two straight lines $2x + y = 0$ and $x - 3y = 0$ intersects at point
 (a) (3, 2) (b) (4, 3) (c) (2, 3) (d) (0, 0)
6. Maximum value of $z = 4x + y$ at (0, 0), (30, 0), (20, 30) and (0, 50) is
 (a) 510 (b) 110 (c) 120 (d) 50
7. The system of equation $x - 2y + z = 3$ and $2x - 5y - z = 4$ has
 (a) unique solution (b) No solution (c) infinitely many solution
8. The system of equations $x + y = 4$ and $x - y = 4$ has
 (a) unique solution (b) infinitely many solution (c) No solution
9. $3 - 2x \geq 7$ implies the inequality
 (a) $x \geq -2$ (b) $-x \leq 2$ (c) $x \leq 2$ (d) $x \leq -2$
10. $2x + 3 \geq 5$ implies the inequality
 (a) $x \geq 1$ (b) $x \leq 1$ (c) $x > 1$ (d) $x < 1$
11. The root of the equation $3x^3 - 2x^2 + 5x - 6 = 0$ is
 (a) 2 (b) 1 (c) 3 (d) 5
12. The fourth root of 260 lies in
 (a) (1, 2) (b) (2, 3) (c) (3, 4) (d) (4, 5)
13. The Newton's second approximation (x_2) of root of $x^5 - x - 1 = 0$ with initial approximation $x_1 = 1$ is
 (a) 1.5 (b) 1.25 (c) 1.75 (d) 1.85
14. The root of equation $f(x) = 0$ always lies in between a and b ($a \neq b$) if
 (a) $f(a) f(b) < 0$ (b) $f(a) f(b) > 0$
 (c) $f(a) > f(b)$ (d) $f(b) < f(a)$
15. The graph of $x > 3$ is
 (a) open half ray (b) closed half ray
 (c) open half plane (d) closed half plane

ANSWERS:

- 1.b 2.a 3.b 4.a 5.d 6.c 7.c 8.a 9.d 10.a
 11.b 12.d 13.b 14.a 15.a

Candidates are required to answer the questions in their own words as far as possible.

Group B

Attempt any SIX questions.

[6×5=30]

11. If a function $f(x)$ is defined as:

$$f(x) = \begin{cases} 3x^2 + 2 & \text{if } x < 1 \\ 2x + 3 & \text{if } x > 1 \\ 4 & \text{if } x = 1 \end{cases}$$

Discuss the continuity of function at $x = 1$.

12. Find the derivative of $\sin 3x$ by using definition.

13. Using L-Hospital's rule evaluate:

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 3x}{1 + 5x^2}$$

14. If demand function and cost function are given by

$P(Q) = 1 - 3Q$ and $C(Q) = Q^2 - 2Q$ respectively, where Q is the quality (number) of the product then find output of the factor for the maximum profit.

15. Evaluate: (a) $\int \frac{dx}{1 - \sin x}$ (b) $\int_0^1 (x^2 + 5) dx$

16. Solve: $\frac{dy}{dx} = \frac{xy + y}{xy + x}$

17. Examine the consistency of the system of equation and solve if possible.

$$x_1 + x_2 - x_3 = 1,$$

$$2x_1 + 3x_2 + 3x_3 = 3$$

$$x_1 - 3x_2 + 3x_3 = 2$$

Group C

Attempt any two questions.

[2×10=20]

18. Define Homogeneous equation and solve the following system of equations using Inverse Matrix Method.

$$-2x + 2y + z = -4$$

$$-8x + 7y - 4z = -47$$

$$9x - 8y + 5z = 55$$

19. State Rolle's Theorem and interpret it geometrically. Verify Rolle's theorem for $f(x) = x^2 - 4$ in $-3 \leq x \leq 3$.

20. Using Composite Trapezoidal Rule, compute $\int_0^2 (2x^2 - 1) dx$ with four intervals. Find the absolute error of approximation from its actual value.